Silent Full-Scale Quantum (SFSQ) computers: a quick primer for investors and funding agencies

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Abstract

A quantum computation occurs when a classically physical programmer sends instructions to a quantum physical system (a.k.a quantum processor) to change its state, and then reads the output of the quantum processor back into the classically physical world, all without any noise or errors in the process. This requires faithful transmission of information across the classical-quantum physical divide, a task that remains a challenge, despite millions of dollars of capital and government funding invested in the development of quantum computers. This sub-optimal financial situation can be mitigated simply by replacing today's ad-hoc, bottom-up methods for designing hardware for quantum computing with a systematic, mathematically robust method for feeding information into a quantum processor (state preparation) and noisy read-out of quantum information (quantum measurement). The mathematical result known as Nash embedding (by Nobel laureate John Nash) is a novel alternative that can take quantum computing beyond the current, first-generation of Noisy Intermediate Scale Quantum (NISQ) processors by designing robust, fault-tolerant, Silent Full Scale Quantum (SFSQ) processors.

1 Introduction

The problem of traversing the classical-quantum physical divide has been effectively depicted in the Marvel Cinematic Universe (MCU) movies, Ant-Man I and II, in recent years. In the first movie of this series, the character Janet van Dyne (the original Wasp) is lost in the quantum world when she shrinks down to quantum scale using technology developed by her husband, Henry Pym. In the MCU, entering the quantum physical world is considered easier than returning back from it to the classically physical world. In reality, however, the former task is just as difficult as the latter. Once in the quantum realm, van Dyne is unable to return due to the disruptive nature of this return journey, a process referred to as *quantum measurement*. The rest of this section will focus primarily on this process.

Quantum measurement (or just measurement) is a mechanism that allows extraction of information from a quantum system into the classical realm. Information is a function of the probability of occurrence of an event, as

- an event with probability 100% is perfectly unsurprising and yields no information,
- the less probable an event is, the more surprising it is and the more information it yields, and
- if two independent events are measured separately, the total amount of information is the sum of the information of the individual events.

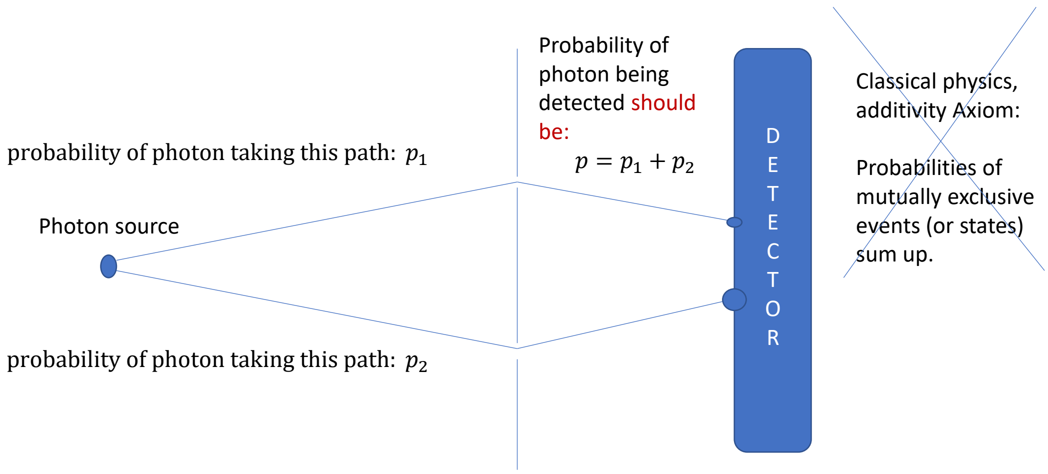

Figure 1: Traditional or classical rules of probability do not apply in the quantum physical realm.

The unique function that satisfies the above axioms for information gathered from an event x is

$$I(x) = -\log\left[\Pr(x)\right] \tag{1}$$

where $\Pr(x)$ is the probability of an event x occurring. The logarithm's base is a matter of choice, with the most popular base being 2, giving rise to the notion of binary digits or *bits* of information. To make the notion of information more accurate, consider a random variable X (such as the outcome of the toss of a coin), which is measured to be in the state x (coin show Heads) with probability $p_X(x)$ (which will equal $\frac{1}{2}$ for the coin showing Heads). Then,

$$I_X(x) = -\log p_X(x). \tag{2}$$

A fundamental characterization of information contained in an event is through the average or expected value of the information. This is known as *entropy*:

$$H(X) = -\sum_x p_X(x) I_X(x) = -\sum_x p_X(x) \log p_X(x). \tag{3}$$

In the case of traversing the classical-quantum divide, the random variable is the state of a quantum system.

But the information that measurement attempts to extract from a quantum system is *quantum information*, that is, information about the quantum system's dynamics within the quantum realm. Quantum information, typically referred to in terms of quantum bits or *qubits*, has properties that are different than classical information, the fundamental difference being that the randomness (probability) associated with it is more general than those associated with classical information. As Feynman shows in one his famous lectures [1], using the famous two-slit experiment exhibited here in Figures 1 and 2, the probability p that a quantum physical object such as a photon will pass through a barrier with two openings is

$$p = p_1 + p_2 + 2\sqrt{p_1 p_2}\cos(\theta_1 - \theta_2) \tag{4}$$

with p_1 being the probability with which the photon will pass through slit 1, and p_2 with which it will pass through slit 2. The variables θ_1 and θ_2 represent *quantum phase*, a feature unique to the quantum

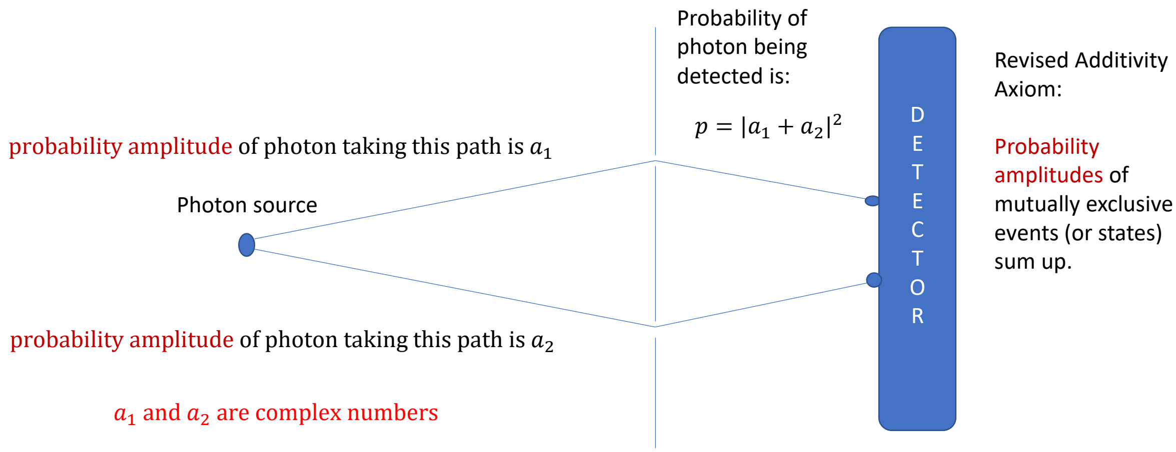

Figure 2: Quantum physical rules of probabilities.

realm. If the photon's quantum state is now measured to determine which slit it passed through, its "quantum-ness" de-coheres or de-phases and the quantum information θ_1 and θ_2 is lost. The probability of the photon passing through the barrier reduces to the anticipated classical probability

$$p = p_1 + p_2. \tag{5}$$

Substituting the expression for p from equation (4) into equations (2) and (3) shows how different and more general quantum information is than classical information. When contrasted with equation (5), it also shows how measurement is fundamentally a disruptive method for (information) traversing the classical-quantum divided. Because measurement does not identify quantum phase, Janet van Dyne could not come back from the quantum to the classical realm. It was not until her husband, Henry Pym, creates the fictional "quantum tunnel" that van Dyne was able to smoothly traverse the classical-quantum divide back to the classical world.

The MCU Ant-man movies have given those seeking to develop the next-gen SFSQ computers two questions to address:

1. How does one load classical information into a quantum processor, without losing its basic features like shape (yes, data has shape!) and rates of change of its dynamics? This is the least Janet van Dyne must have managed to do in Ant-man, using the fictional Pym particle, with the classical information that described her physical presence in the classical world, thus allowing her to travel into the quantum world.

   SFSQ processors should be designed to allow faithful mapping, in the above context, of classical data points into quantum data points.

2. How does one extract classical information from a quantum processor, that is, read the quantum information into the classical world, faithfully? Quantum measurement does not achieve this; it ignores at least the relative quantum phase (the distance between two probability amplitudes). Otherwise, van Dyne would not have got stuck in the quantum realm for all these years. Pym created the fictional quantum tunnel for this faithful extraction.

SFSQ processors should have similar functionality.

While a physical mechanism to addresses the two questions above is not understood yet, a mathematical mechanism that is the correct first step toward this understanding exists, and is well understood. This is the Nash embedding.

2 Nash embedding - the mathematical mechanism

John Nash was an eminent mathematician in the 1950s working on problems of pure mathematical interest as well as mathematics of games. He was struck by mental illness that led to a forty year hiatus from serious mathematical research and recognition. This changed in 1993 when he was awarded the Nobel prize in Economics for his formulation of a solution concept for general non-cooperative games, the famous Nash equilibrium. A nice biopic of Nash came out in 2001, titled *A Beautiful Mind*, showcasing his genius but trouble mind. However, this movie almost completely ignored Nash's brilliant work in pure mathematics, giving it only a sentence long mention near the end of the movie.

This work of Nash [2] showed that it is possible to embed a *Riemannian manifold*, which is what the quantum physical realm is, into the *Euclidean space*, which is what the classical physical realms is, all the while preserving lengths of data points. In crude layman terms, Nash showed that it is possible to "merge" a contorted space, along with any contours and curves drawn on it (non-Euclidean geometry), into a *flat* Euclidean space (like an infinite piece of paper or in fact our 3-dimensional physical world of every day experience) so that the lengths of the contours on the space do not change in its Euclidean image. Formally:

Nash embedding theorem: For every d-dimensional Riemannian manifold M, there exists an isometric (distance preserving) embedding of M into $\mathbb{R}^m$ for a <u>suitably large</u> m.

The space $\mathbb{R}^m$ represents Euclidean space of dimension m. When $m = 1$, this space is a one-dimensional line, $\mathbb{R}$. For $m = 2$, it is a two dimensional plane $\mathbb{R}^2$ which, when co-ordinateized, is called the Cartesian plane. For $m = 3$, the Euclidean space is the space of our everyday physical experience, $\mathbb{R}^3$. For $m > 3$ we cannot visualize the Euclidean space directly; however, it is possible to write equations and create geometric projections of high dimensional Euclidean space into lower dimensional one's (think of the shadow of a sphere onto the plane being an ellipse). See Figure 3.

The word merge is replaced in the theorem statement with the more technical word "embedding". I will explain this word in just a bit. But first, What does this seemingly esoteric result have to do with traversing the classical-quantum divide smoothly, and the emerging technology of quantum computing and communication? A lot.

Embedding theorems like those of Nash were formulated to answer questions about the effects on the mathematical properties of a space when it was merged with a space with different properties. In both mathematics and physics, properties of topology (shape of a space), differentiability (optimization of dynamical processes in the space), and geometry (lengths, distance between points) are considered fundamental as physical observations can be understood with respect to these properties. Over time, it has been observed that the quantum physical realm has the topology, differential structure, and geometry of a particular Riemannian manifold that is distinct that the Euclidean space of classical physics. A simple example of this distinction is the sphere in 3 dimensional space - the Riemannian manifold describing one qubit - in contrast to the two element set $\{0, 1\}$ inside the Euclidean space describing a single bit of classical information. Hence, embedding theorems are crucial to understand how to effectively transmit information across the classical-quantum physical divide.

Refer back to equation (4) that contains relative phase information $\theta_1 - \theta_2$. Geometrically, relative

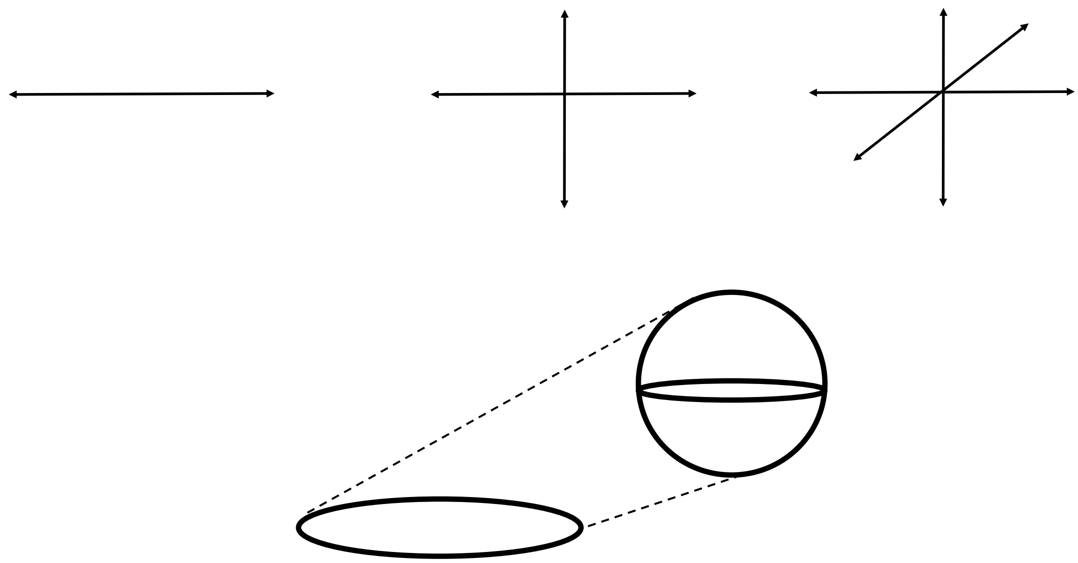

Figure 3: First row figures, from left to right, are representations of one, two, and three dimensional Euclidean spaces. The figure in the second row shows a projection of a sphere onto the two dimensional plane, resulting in an ellipse.

phase is the distance between the two probability amplitudes, a_1 and a_2, in the curvy space of a qubit. Quantum measurement can be viewed as an attempt to merge the quantum space with the Euclidean space with the goal of extracting quantum information into the classical world. Unfortunately, this process is not faithful.

But embeddings are, meaning that a Riemannian manifold can be identified with a subset of the Euclidean space, in a way that is one-to-one and onto and that preserves topology (shape: points close to each other stay close) and differentiability (rates of change: optimization). The preservation of these mathematical properties will preserve corresponding fundamental features of quantum information when it is extracted into the classical world.

Nash embedding is even better, being faithful up to lengths and distances as well. This means that Nash embedding, when used as the mechanism for sending information across the classical-quantum physical divide, will not only preserve topological and differential features of information but also preserve distance, or relative phase. The fictional Pym particle is a physical mechanism implementing Nash embedding.

Next, let's address the two questions raised in section 1 in light of the preceding discussion of Nash embedding.

2.1 Loading classical information into a quantum processor

The problem of loading or encoding classical information into a quantum processor is referred to as state preparation in quantum computing. In current theory and practice in quantum computing circles, it is done by way of an ad-hoc identification of classical data points with the observable states of a quantum system (elements of an orthonormal basis of the quantum system), and then a *unitary* action is performed on this data to prepare an initial state of the quantum system that encodes the quantized classical information.

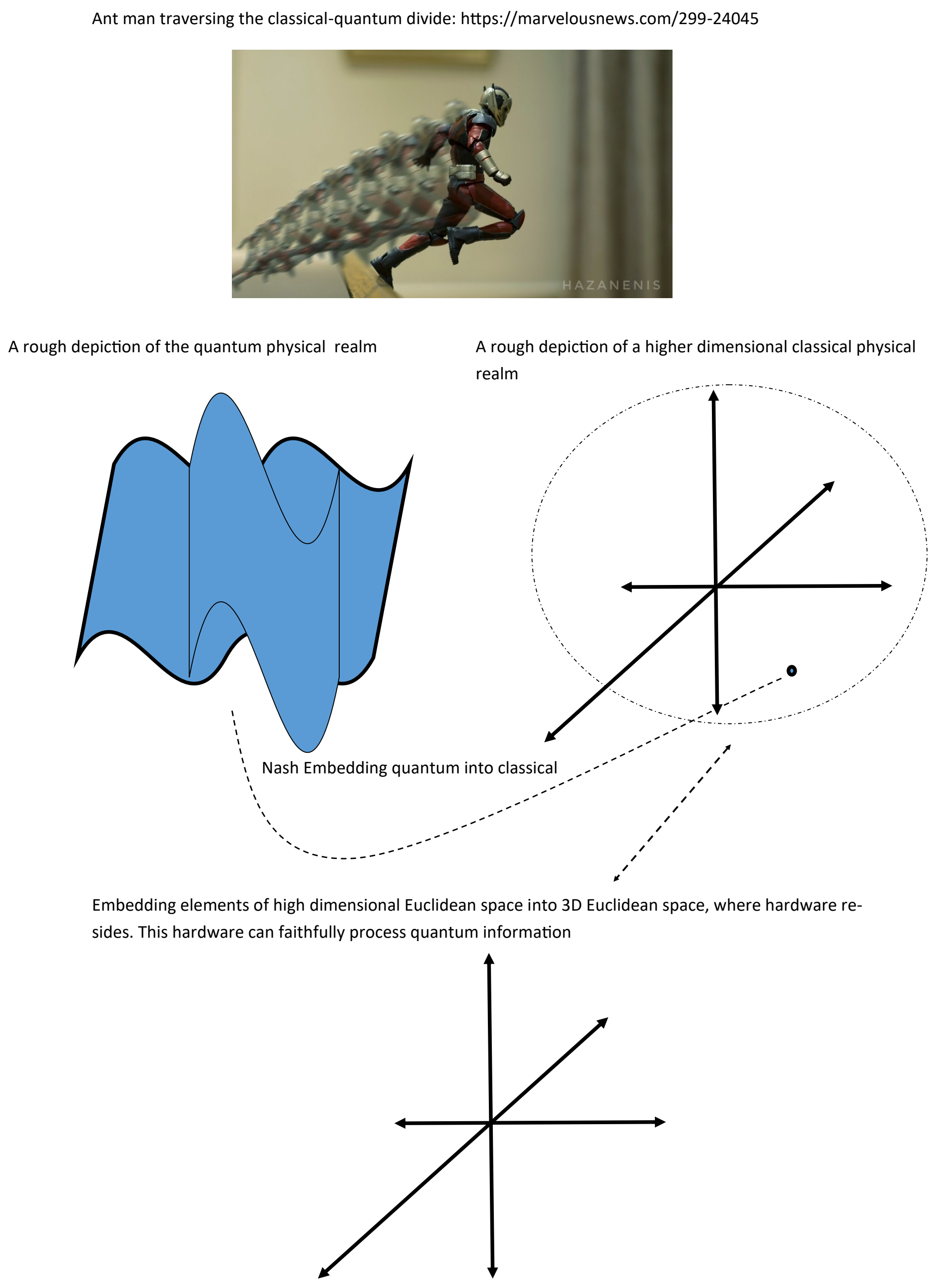

Figure 4: First row depicts the fantastical idea of Ant Man from the MCU traversing the classical-quantum divide. Second row is a high-level, cartoon depiction of the quantum realm, the Riemannian space of qubits, being Nash embedded (not measured) into the Euclidean realm of bits, albeit in high dimensions. Finally, using established graph-theoretic techniques from the days of VLSI fabrication, tangible hardware is developed in the three dimensional world we live in.

Such ad-hoc protocols to initializing a quantum processor with classical information, (e.g., Flexible Representation of Quantum Images (FRQI) and other protocols [3] in quantizing classical information to represent images in the quantum realm) are motivated by the desire to reproduce transformation of classical information inside the quantum realm. But these identifications do not necessarily preserve the shape of the classical data inside the quantum realm (points that were close in the classical world do not necessarily stay close to their image in the quantum realm, and distances can contort) because unlike quantum information, classical information can transform non-unitarily and in fact *non-linearly*! Likewise for rates of change.

Nash's embedding on the other hand is a continuous, one-to-one and onto map of the classical realm into the quantum that preserves topology, rates of change, and distances between points. This means that not only do classical data points close to each other stay close to each other when quantized under Nash's scheme, but the exact distances between them stay identical! Finally, rates of change of functions of classical data points also stay the same under this quantization scheme, allowing one to do Calculus (optimization) faithfully when loading a quantum computer with classical information.

2.2 Reading out quantum information into the classical world

We spent considerable space in section 1 discussing quantum measurement as the preferred physical mechanism of the past and present for the extraction of quantum information into the classical world. As noted in details in the introductory section, measurement does not faithfully reproduce relative quantum phase information into the classically world. Since relative phase is the distance between two states of a quantum object, not having it mapped faithfully into the corresponding classical data points in the read-out produces errors and distortion in the resulting classical data.

As in the previous section, concerns about transformation of quantum data, which is necessarily both linear and unitary, are valid here as well. Does measurement preserve the latter when the quantum information is mapped into the classical real? Do points close to each other in the quantum realm remain close in their classical image? No, it does not.

Let's focus on unitarity of quantum physical transformations: this means that in the quantum realm, distances between qubits states are preserved when they are transformed. But this is not the case in classical information, where bit states are transformed without any such properties. To preserve this property of quantum information when it is read out into the classical world, classical computing requires to be *reversible*, something that is theoretically well understood, but the practical implementation of which remains elusive, meaning that no hardware exists that can implement classical reversible computing.

So while quantum computing is naturally reversible, classical computing requires further study to be made so at the engineering level. Being an isometric embedding, Nash embedding in fact shows that the faithful classical implementation of quantum computing is reversible. See Figure 5. Therefore, hardware for quantum computing in the Euclidean realm should be designed as reversible computing, followed by implementing well-known graph-theoretic considerations for actual fabrication.

3 Conclusion

This document serves as a quick guide for those looking to invest in the development of hardware for properly quantum computers that can function beyond the current NISQ computers. These "Silent, Full Scale Quantum" (SFSQ) computers will be able to fulfil the promise of quantum computing by implementing Shor's algorithm [4] for factoring numbers trivially, a task still considered impossible on conventional computers. SFSQ computers will also be able to implement Grover's algorithm [5] properly, delivering the promised speed up in searching unstructured databases that is proven to be impossible for conventional computing. The message of this report is to invest in efforts that study Nash

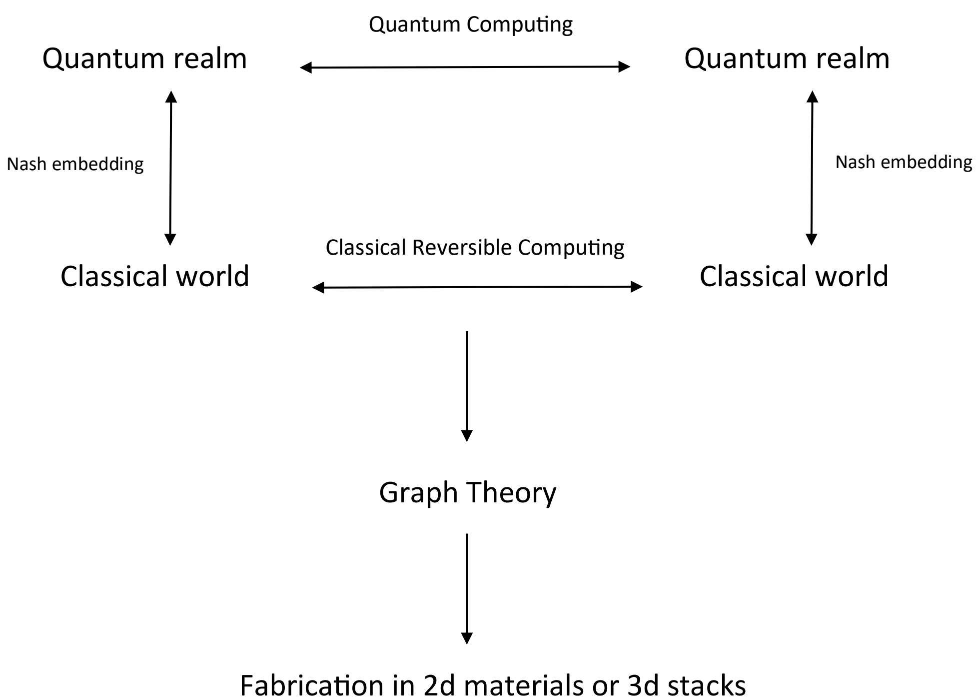

Figure 5: Layout for executing SFSQ computing. Quantum computing occurs faithfully in the quantum realm, while reversible computing happens faithfully in the classical world. Nash embedding allows a faithful transmission of information across the classical-quantum physical divide. Current approaches to this transmission are incomplete and will not produce robust and fault-tolerant quantum computing.

embedding as a method for designing physical mechanisms that can faithfully transmit information between the classical-quantum physical divide for robust, fault-tolerant quantum computing that will make SFSQ computers a reality.

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